

2)Internet trick - FIDO problem

<http://www.digicc.com/fido/>

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- a) Do you see how Fido always guesses the number?
- b) To explain why it always works involves reviewing *casting out nines*.
 - Are you familiar with the casting out nines technique for checking addition & multiplication problems?

It uses the idea of a **digital root**, Every number has a digital root— found by adding up the digits composing a number, until you get a single digit. e.g. $756 \rightarrow 7+5+6 \rightarrow 18 \rightarrow 9$ is its digital root.

casting out nines gets its name from the fact that since a 9 adds no value to the final answer of the digital root, we might as well save time and not bother to include them in the adding process. How can we prove this? 19, 29, 39, 49, 59, 69, 79, 89, \rightarrow all leave these numbers digital roots unchanged, so we toss them out before adding.

Hence, its name casting out nine.

What about 0? Do they change the digital root? If not ignore them also.

Note: casting out nines does not always work to check + and x. The method does not pick-up an error if you accidentally reverse the digits in an answer, or you accident replaced a 9 for a 0 or vice versa, in an answer.

NOW LETS APPLY casting out nines to the FIDO problem

- 1) Give me a 4 digit number with lots of different digits. 4761
- 2) Jumble-up its digits: 7146, and subtract the larger from the smaller. **Both numbers have the same digital root, so when they are subtracted the result is 0.** The difference must have this result according to casting out by nines.

$$7146 - 4761 = 2385$$

- 3) Draw a circle around one of the digits. Don't pick 0. (Why? Because 0 and 9 value can't be distinguished—neither affects the digital root of a number.) Suppose we choose 8. $23\textcircled{8}5 \rightarrow$ we know its digital root must total 0, that is it's the number 9 (which was cast out). So all you have to do is determine what must be added to $2+3+?+5$ to make a final digital root of 9. $2+3+5 = 10=1$, so it must be a 8.

In the FIDO problem we were asked to do a subtraction.

$$7146 - 4761 = 2385$$

dig.roots

$$9 - 9 \quad 10+\underline{8} \text{ is } 18 \text{ which is a } 9, \text{ which is equivalent to } 0 \text{ or } 9.$$